Metric Properties of generalizations of Thompson's Group F

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Multiplying tree-pair diagrams in $F(n)$

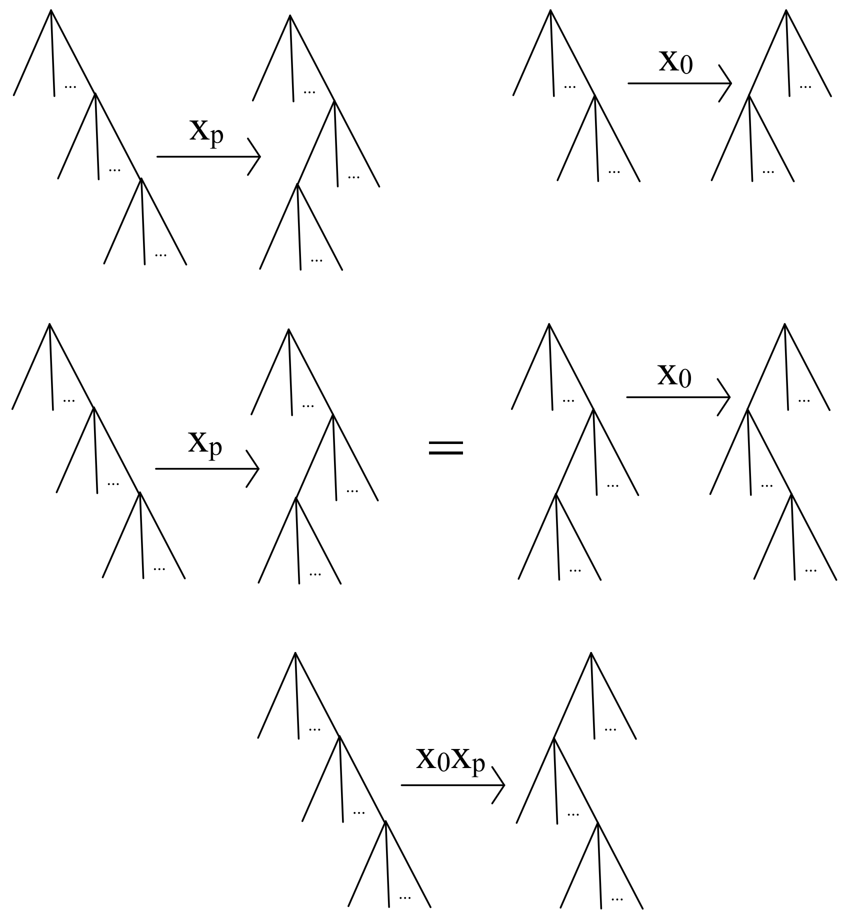


Figure 1: Composition of two elements of $F(n)$

Multiplying tree-pair diagrams in $F(n_1, ..., n_k)$

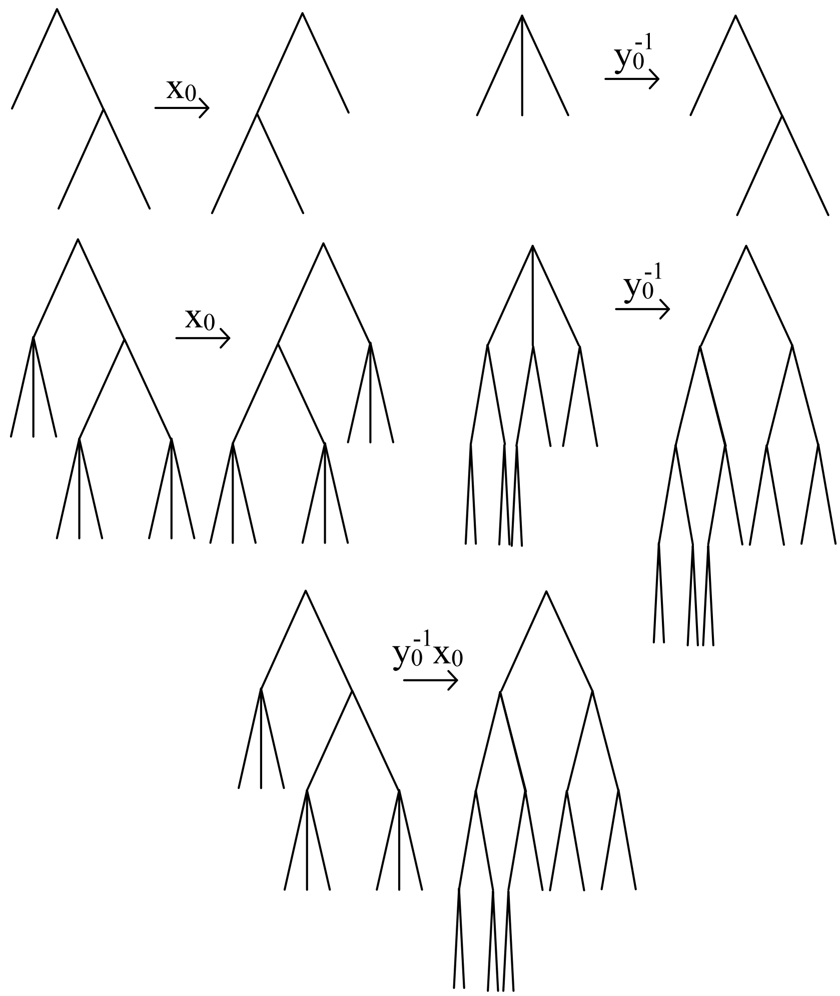
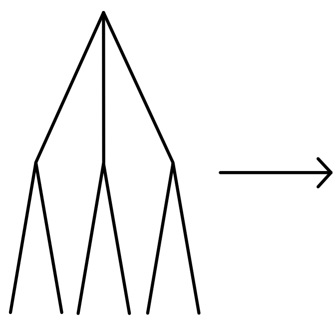
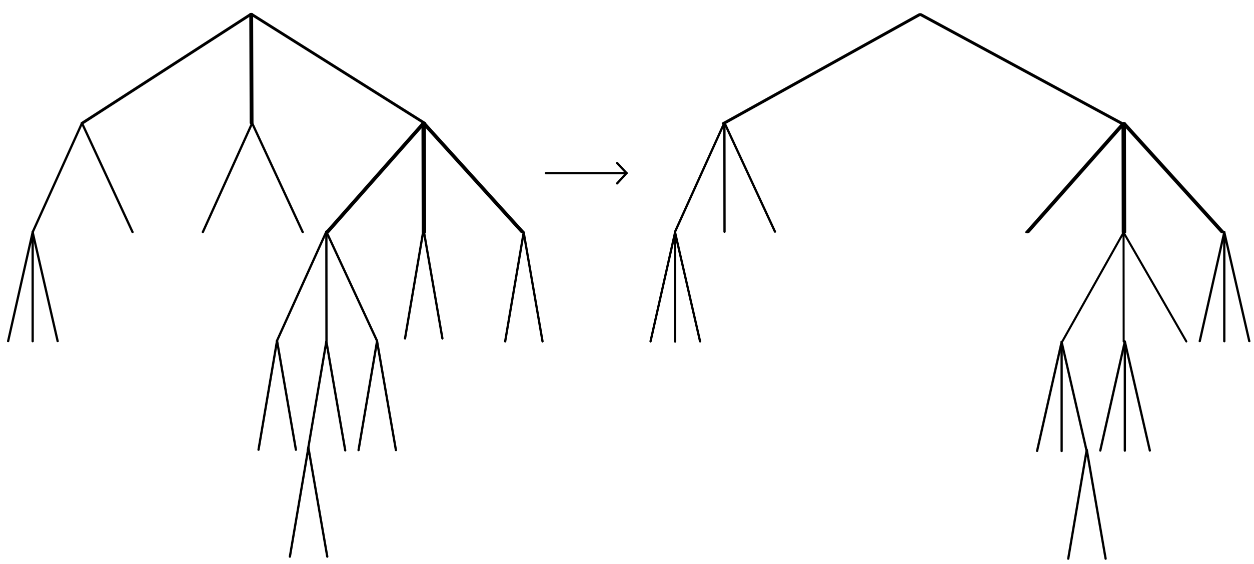


Figure 2: Composition of two elements of $F(2, 3)$

Representatives of the identity

Tree-pair diagrams representing the identity in $F(n)$ will always consist of two identical trees. This is not the case in $F(n_1, ..., n_k)$.





Minimal tree-pair diagram representatives may not be unique

Minimal tree-pair diagram representatives of $F(n)$ are unique. This is not the case in $F(n_1, ..., n_k)$.

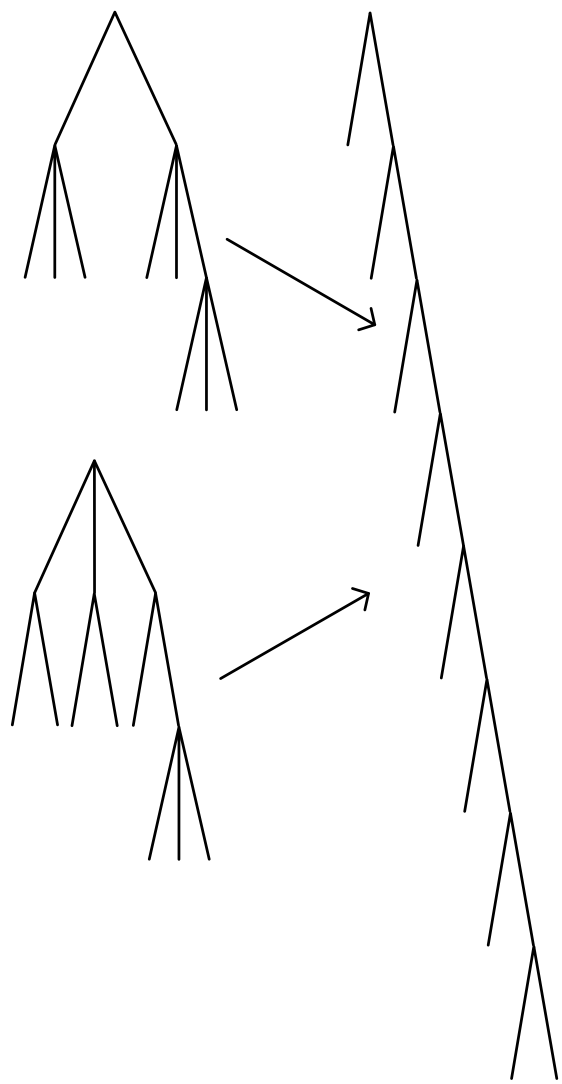


Figure 3: Two equivalent but distinct minimal tree-pair diagrams representing an element of $F(2, 3)$

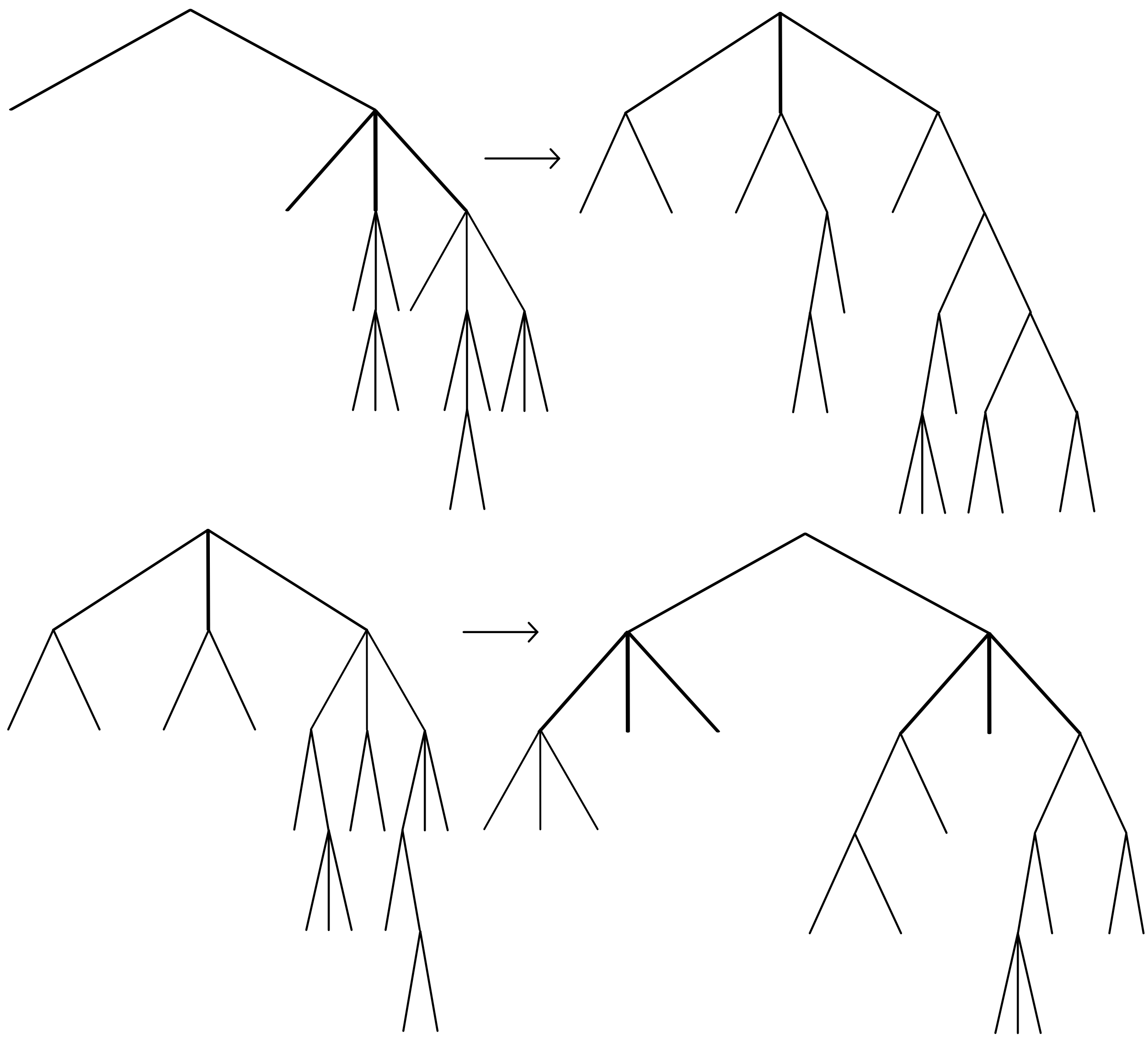


Figure 4: Two equivalent but distinct minimal tree-pair diagrams representing another element of $F(2,3)$

To get to the minimal tree-pair diagram, we may have to add carets

Minimal tree-pair diagram representatives of $F(n)$ can always be obtained solely by caret removal. Whereas in $F(n_1, ..., n_k)$, we may even need to add carets in order to obtain a minimal tree-pair diagram from a given tree-pair diagram.

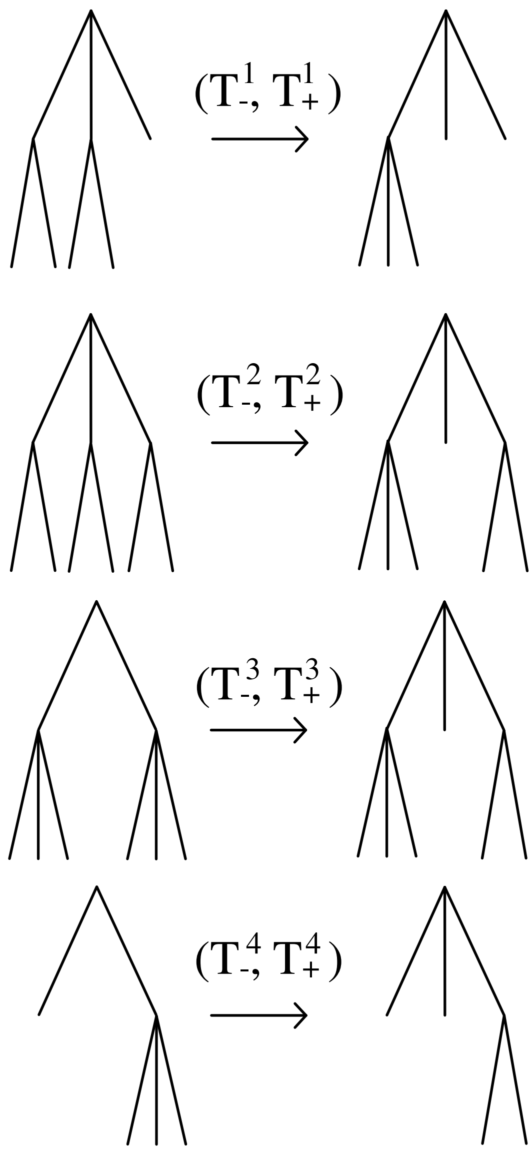
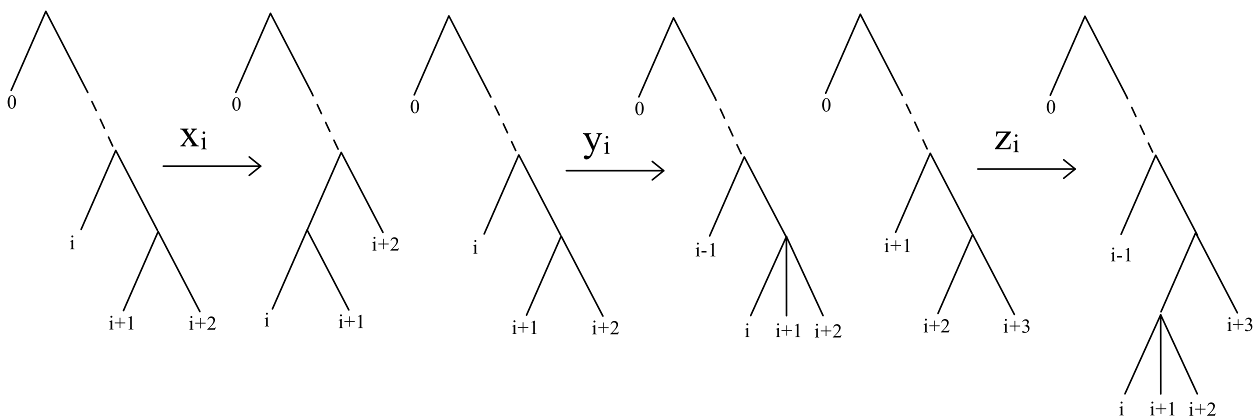


Figure 5: (T^1_-, T^1_+) is a (2,3)–ary tree-pair diagram which must have carets added to it in order to be transformed into its equivalent minimal tree-pair diagram (T^4_-, T^4_+)

Standard infinite presentation (Stein)

Generators:

$$\{x_0, x_1, ..., y_0, y_1, ..., z_0, z_1, ...\}$$



Relators:

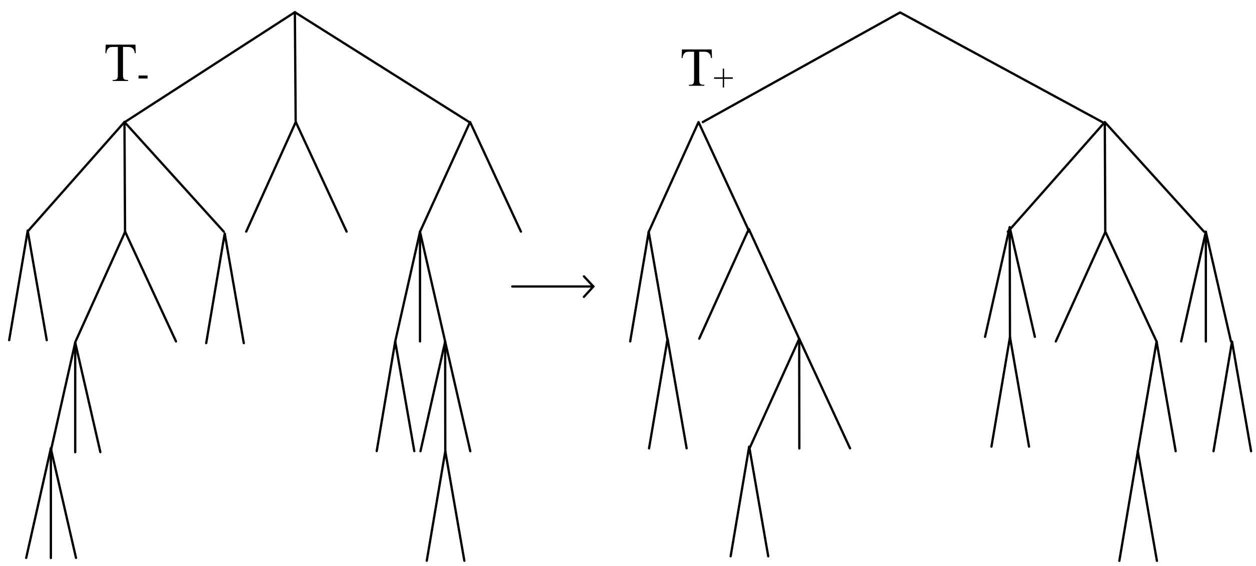
1. $x_j x_i = x_i x_{j+1}$
2. $y_j x_i = x_i y_{j+1}$
3. $z_j x_i = x_i z_{j+1}$
4. $x_j z_i = z_i x_{j+2}$
5. $y_j z_i = z_i y_{j+2}$
6. $z_j z_i = z_i z_{j+2}$

for $i < j$ and

1. $y_{i+1} z_i = y_i x_{i+1} x_i$
2. $x_i z_{i+1} z_i = z_i x_{i+2} x_{i+1} x_i$

for all i.

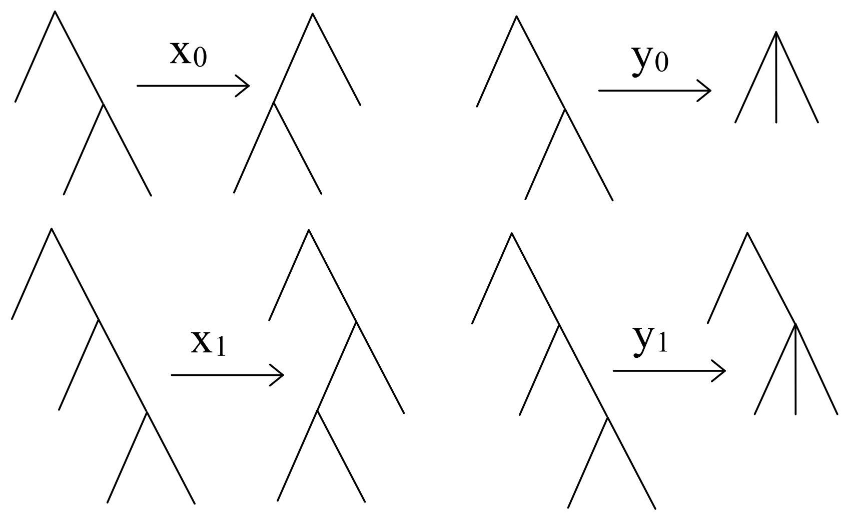
Normal Form Example



Standard finite presentation (Stein)

Generators:

$$\{x_0, x_1, y_0, y_1\}$$



Relators:

1. $x_2x_0 = x_0x_3$
2. $x_3x_1 = x_1x_4$
3. $y_2x_0 = x_0y_3$
4. $y_3x_1 = x_1y_4$
5. $x_1z_0 = z_0x_3$
6. $x_2z_1 = z_1x_4$
7. $y_1z_0 = z_0y_3$
8. $y_2z_1 = z_1y_4$
9. $x_0z_1z_0 = z_0x_2x_1x_0$
10. $x_1z_2z_1 = z_1x_3x_2x_1$

where $x_3 = x_1^{-1}x_2x_1$, $x_4 = x_2^{-1}x_3x_2$, $y_3 = x_1^{-1}y_2x_1$, $y_4 = x_2^{-1}y_3x_2$, $z_0 = y_1^{-1}y_0x_1x_0$, $z_1 = y_2^{-1}y_1x_2x_1$, and $z_2 = y_3^{-1}y_2x_3x_2$.